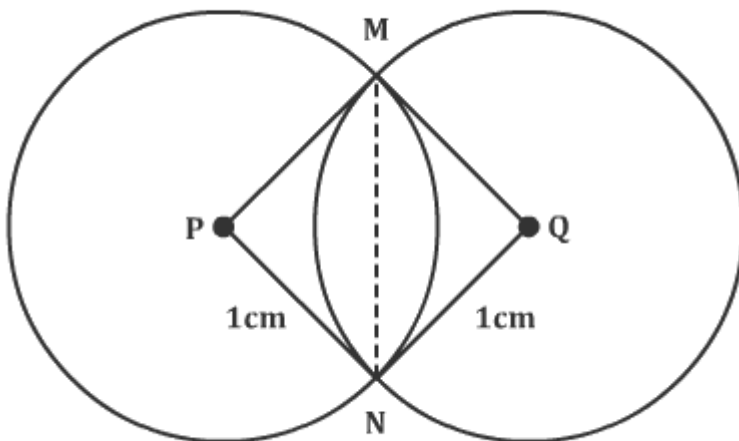


**Solutions**

1.  $x = 16^3 + 17^3 + 18^3 + 19^3$   
 $= (16^3 + 19^3) + (17^3 + 18^3)$   
 $= (16 + 19)(16^2 + 16 \times 19 + 19^2) + (17 + 18)(17^2 + 17 \times 18 + 18^2)$   
 $= 35 \times (\text{an odd number}) + 35 \times (\text{another odd number}) = 35 \times (\text{an even number})$   
 $= 35 \times (2k) \dots (k \text{ is a positive integer})$   
 $\therefore x = 70k$   
 $\therefore x$  is divisible by 70.  
 Remainder when  $x$  is divided by 70 = 0  
 Hence, option 1.

2. The change in the amount of chemical in each tank after every minute is as follows:  
 A:  $-20 - 10 + 90 = 60$   
 B:  $-100 + 110 + 20 = 30$   
 C:  $-50 - 90 + 100 = -40$   
 D:  $-110 + 10 + 50 = -50$   
 Since tank D loses the maximum amount of chemical in a minute, it will be emptied first.  
 Let  $n$  minutes be the time taken by tank D to get empty.  
 $\therefore 1000 - 50n = 0$   
 $\therefore n = 20$  minutes  
 Hence, option 3.

3.



Let the two circles with centres P and Q intersect at M and N.  
 Quadrilateral PQMN is a square.  
 $m\angle MPN = m\angle MQN = 90^\circ$   
 The area common to both the circles =  $2(\text{Area of sector P-MN} - \text{Area of } \Delta PMN)$

$$= 2[(90/360 \times \pi \times 1^2) - (1/2 \times 1^2)]$$

$$= \pi/2 - 1$$

Hence, option 2.

4. Let  $r$  be the radius of the circular tracks.

Length and breadth of the rectangular track are  $4r$  and  $2r$  respectively.

Length (perimeter) of the rectangular track =  $12r$

Length of the two circular tracks (figure of eight) =  $4\pi r$

If A and B have to reach their starting points at the same time,

$$\frac{12r}{a} = \frac{4\pi r}{b}$$

(where  $a$  and  $b$  are the speeds of A and B respectively)

$$\therefore \frac{b}{a} = \frac{4\pi}{12}$$

$$\therefore (b - a) \times 100/a = 0.047 \times 100$$

$$= 4.7\%$$

Hence, option 4.

5. Let there be  $g$  girls and  $b$  boys.

Number of games between two girls =  ${}^g C_2$

Number of games between two boys =  ${}^b C_2$

$$\therefore g(g - 1)/2 = 45$$

$$\therefore g^2 - g - 90 = 0$$

$$\therefore (g - 10)(g + 9) = 0$$

$$\therefore g = 10$$

Also,

$$b(b - 1)/2 = 190$$

$$\therefore b^2 - b - 380 = 0$$

$$\therefore (b + 19)(b - 20) = 0$$

$$\therefore b = 20$$

$$\therefore \text{Total number of games} = (g + b)C_2 = {}^{30}C_2 = 435$$

$$\therefore \text{Number of games in which one player is a boy and the other is a girl} = 435 - 45 - 190 = 200$$

Hence, option 1.

6. Ram and Shyam run a race between points A and B, 5 km apart. Ram starts at 9 a.m. from A at a speed of 5 km/hr, reaches B, and returns to A at the same speed. Shyam starts at 9:45 a.m. from A at a speed of 10 km/hr, reaches B and comes back to A at the same speed.

Ram starts at 9:00 am and Shyam starts at 9:45 am from A.

Ram reaches B at 10:00 am (as his speed is 5km/hr and the distance between A and B is 5km)

When Ram reaches B, Shyam is  $15/60 \times 10 = 2.5$  km away from A.

Ram meets Shyam  $(2.5 \times 60)/(10 + 5)$  minutes after 10:00 a.m. i.e., at 10:10 a.m.

Shyam reaches B at 10:15 a.m.

At 10:15 a.m., Ram is  $(15/60) \times 5 = 1.25$  km away from B.

Shyam overtakes Ram in  $1.25/(10 - 5) = 0.25$  hrs = 15 minutes after 10:15 am i.e. at 10:30 a.m.

Hence, option 2.

7. Shyam overtakes Ram at 10:30 a.m.

Hence, option 2.

8.

$$R = \frac{30^{65} - 29^{65}}{30^{64} + 29^{64}}$$

$$\therefore a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1})$$

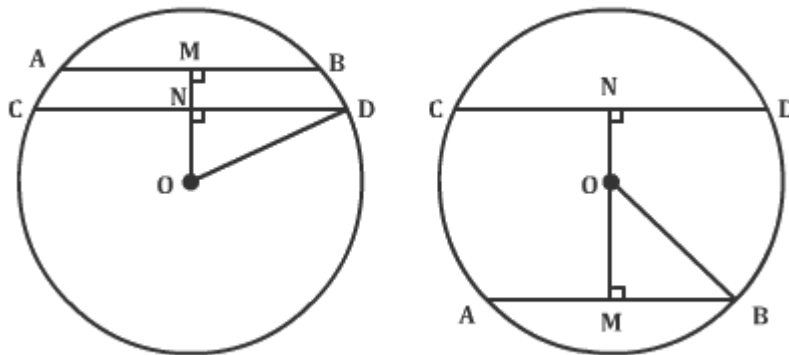
$$\therefore R = \frac{(30 - 29)[30^{64} + (30^{63} \times 29) + \dots + 29^{64}]}{30^{64} + 29^{64}}$$

$$\therefore 30^{64} + 30^{63} \times 29 + \dots + 29^{64} > 30^{64} + 29^{64}$$

$$\therefore R > 1$$

Hence, option 4.

9.



The two chords AB and CD can be on the same side or the opposite sides of the centre O.

Let M and N be the midpoints of AB and CD.

$\therefore$  MN is the distance between the two chords.

MB = 12 cm and ND = 16 cm

OM and ON are perpendicular to AB and CD respectively.

$\therefore$   $ON^2 = 20^2 - 16^2$  (By Pythagoras theorem)

$\therefore$  ON = 12 cm

Similarly, OM = 16 cm

Case 1: AB and BC are on the same side of the centre.

$$MN = OM - ON = 4 \text{ cm}$$

Case 2:  $MN = OM + ON = 28 \text{ cm}$

Hence, option 4.

**10.** We have,  $x^2 = y^2$  and  $(x - k)^2 + y^2 = 1$

Solving the two equations simultaneously, we get,

$$(x - k)^2 + x^2 = 1$$

$$\therefore x^2 - 2kx + k^2 + x^2 = 1$$

$$\therefore 2x^2 - 2kx + (k^2 - 1) = 0$$

If this equation has a unique solution for  $x$ , then discriminant = 0

$$\therefore 4k^2 - 8(k^2 - 1) = 0$$

$$\therefore 8 - 4k^2 = 0$$

$$\therefore k^2 = 2$$

$$\therefore k = \pm\sqrt{2}$$

Since  $k$  is positive the other solution is ruled out

$$\therefore k = \sqrt{2}$$

Hence, option 3.

**11.**  $p = (1 \times 1!) + (2 \times 2!) + (3 \times 3!) + (4 \times 4!) + \dots + (10 \times 10!)$

$$\text{Now, } n \times n! = [(n + 1) - 1] \times n! = (n + 1)! - n!$$

$$\therefore p = 2! - 1! + 3! - 2! + 4! - 3! + 5! - 4! + \dots + 11! - 10!$$

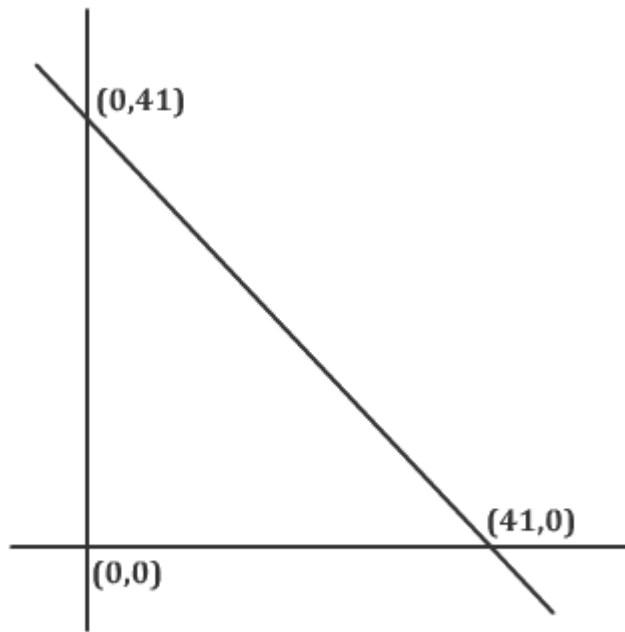
$$\therefore p = 11! - 1! = 11! - 1$$

$$\therefore p + 2 = 11! + 1$$

$\therefore p + 2$  when divided by  $11!$  leaves a remainder of 1.

Hence, option 4.

12.



The points satisfying the equations  $x + y < 41$ ,  $y > 0$ ,  $x > 0$  lie inside the triangle.

Integer solutions of  $x + y < 41$  can be found as follows.

If  $x + y = 40$

$(x, y)$  (1, 39), (2, 38), ..., (39, 1) ... (39 solutions)

If  $x + y = 39$

(1, 38), (2, 37), ..., (38, 1) ... (38 solutions)

If  $x + y = 38$ , we get 37 solutions and so on till  $x + y = 2$  ... (1 solution)

Thus there are  $39 \times 40 / 2 = 780$  integer solutions to  $x + y < 41$

The number of points with integer coordinates lying inside the circle = 780

Hence, option 1.

13. Let  $A = 100x + 10y + z$  ( $x \neq 0$ ,  $x, y, z$  are single digit numbers)

$$\therefore B = 100z + 10y + x$$

$$\therefore B - A = 99(z - x)$$

As  $(B - A)$  is divisible by 7 and 99 is not,  $(z - x)$  is divisible by 7

$\therefore z$  and  $x$  can have values (8, 1) or (9, 2)

$y$  can have any value from 0 to 9.

$$A = 1y8 \text{ or } 2y9$$

$\therefore$  Lowest possible value of  $A$  is 108 and the highest possible value of  $A$  is 299.

Hence, option 2.

14.  $a_1 = 1$

$$a_{n+1} = 4n + 3a_n - 2$$

$$a_2 = 4 - 2 + 3(1) = 5 = 3^2 - 1$$

$$a_3 = 4(2) + 3(5) - 2 = 21 = 3^3 - 6$$

$$a_4 = 4(3) + 3(21) - 2 = 73 = 3^4 - 8$$

$$\therefore a_n = 3^n - 2(n)$$

$$\therefore a_{100} = 3^{100} - 200$$

Hence, option 3.

**15.** Let O and E represent odd and even digits respectively.

$\therefore$  S can have digits of the form  
 O \_ O \_ E or O \_ E \_ O or E \_ O \_ O

Case 1: O \_ O \_ E

The first digit can be chosen in 3 ways out of 1, 3 and 5

The third can be chosen in 2 ways.

The fifth digit can be chosen in 2 ways after which the second and fourth digits can be chosen in 2 ways.

$\therefore$  There are  $3 \times 2 \times 2 \times 3 = 24$  ways in which this number can be written. 12 out of these ways will have 2 in the rightmost position and 12 will have 4 in the rightmost position.

$\therefore$  The sum of the rightmost digits in Case 1 =  $(12 \times 2) + (12 \times 4) = 72$

Case 2: O \_ E \_ O

This number can again be written in 24 ways.

8 out of these ways will have 1 in the rightmost position, 8 will have 3 in the rightmost position and 8 will have 5 in the rightmost position.

Thus the sum of the rightmost digits in Case 2 =  $(8 \times 1) + (8 \times 3) + (8 \times 5) = 72$

Case 3: E \_ O \_ O

This number can also be written in 24 ways.

As in Case 2, 8 out of these ways will have 1 in the rightmost position, 8 will have 3 in the rightmost position and 8 will have 5 in the rightmost position.

$\therefore$  The sum of the rightmost digits in Case 3 =  $(8 \times 1) + (8 \times 3) + (8 \times 5) = 72$

$\therefore$  The sum of the digits in the rightmost position of the numbers in S =  $72 + 72 + 72 = 216$

Hence, option 2.

**16.**  $30^{2720} = 3^{2720} \times 10^{2720}$

The rightmost non zero digit of  $30^{2720}$  will be the digit in the unit's place of  $3^{2720}$ .

3's power cycle is 3, 9, 7, 1 and cyclicity is 4.

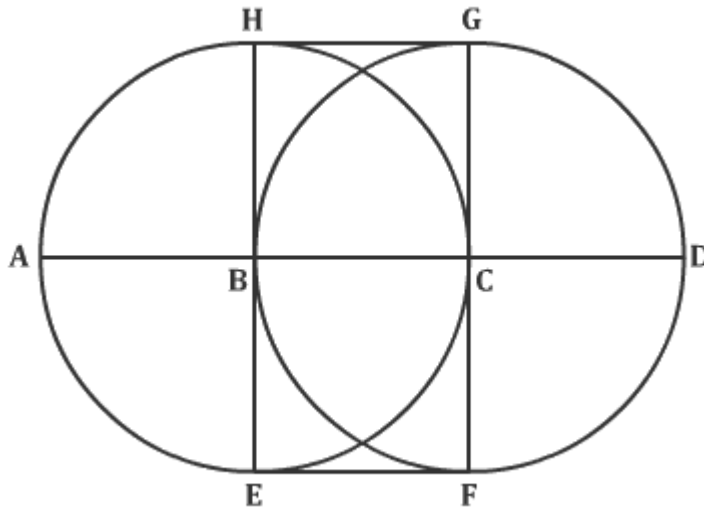
$$2720 = 680 \times 4$$

$\therefore$  The digit in the unit's place of  $3^{2720}$  is 1.

$\therefore$  The rightmost non-zero digit of  $30^{2720}$  is 1.

Hence, option 1.

17.



The ant will not go into the circles with centers B and C and radius = 1 m  
 The minimum distance that the ant has to traverse = the distance of the path A-H-G-D

$$HG = 1\text{ m}$$

$$AH = GD = \frac{1}{4} \times \text{Circumference of Circle} = \frac{\pi}{2}$$

$$AH + GD = \pi \text{ m}$$

$\therefore$  The ant must traverse  $1 + \pi \text{ m}$

Hence, option 2.

18.

$$\begin{aligned} \log_x \left( \frac{x}{y} \right) + \log_y \left( \frac{y}{x} \right) &= \frac{\log x - \log y}{\log x} + \frac{\log y - \log x}{\log y} \\ &= 1 - \log_x y + 1 - \log_y x \\ &= 2 - \log_x y - \log_y x \\ &= 2 - (\log_x y + \log_y x) \end{aligned}$$

As  $x \geq y$  and  $y > 1$ ,

$$\log_y x \geq 1 \text{ and } \log_x y \leq 1$$

$$\log_y x + \log_x y > 1$$

$$\therefore 2 - (\log_x y + \log_y x) < 1$$

$$\therefore \log_x \left( \frac{x}{y} \right) + \log_y \left( \frac{y}{x} \right) \neq 1$$

Hence, option 4.

19.  $n$  can be a 2 digit or a 3 digit number.

Case (I)

Let  $n$  be a 2 digit number.

Let  $n = 10x + y$ , where  $x$  and  $y$  are non-negative integers,

$$P_n = xy \text{ and } S_n = x + y$$

Now,  $P_n + S_n = n$

$$\therefore xy + x + y = 10x + y$$

$$\therefore xy = 9xy = 9$$

There are 9 two digit numbers (19, 29, 39, ..., 99) for which  $y = 9$

Case (II)

Let  $n$  be a 3 digit number.

Let  $n = 100x + 10y + z$ , where  $x, y$  and  $z$  are non-negative integers,

$$P_n = xyz \text{ and } S_n = x + y + z$$

Now,  $P_n + S_n = n$

$$xyz + x + y + z = 100x + 10y + z$$

$$\therefore xyz = 99x + 9y$$

$$\therefore z = 99/y + 9/x$$

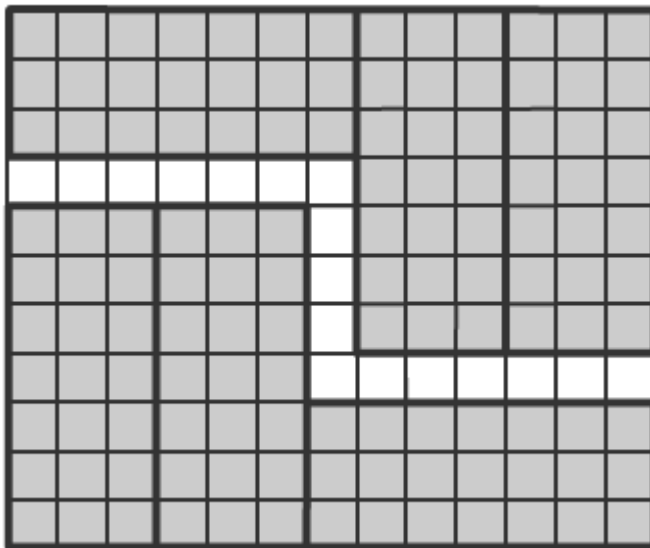
From the above expression,  $0 < x, y < 9$

But, we cannot find any value of  $x$  and  $y$ , for which  $z$  is a single digit number.

$\therefore$  There are no 3 digit numbers which satisfy  $P_n + S_n = n$

Hence, option 4.

20.

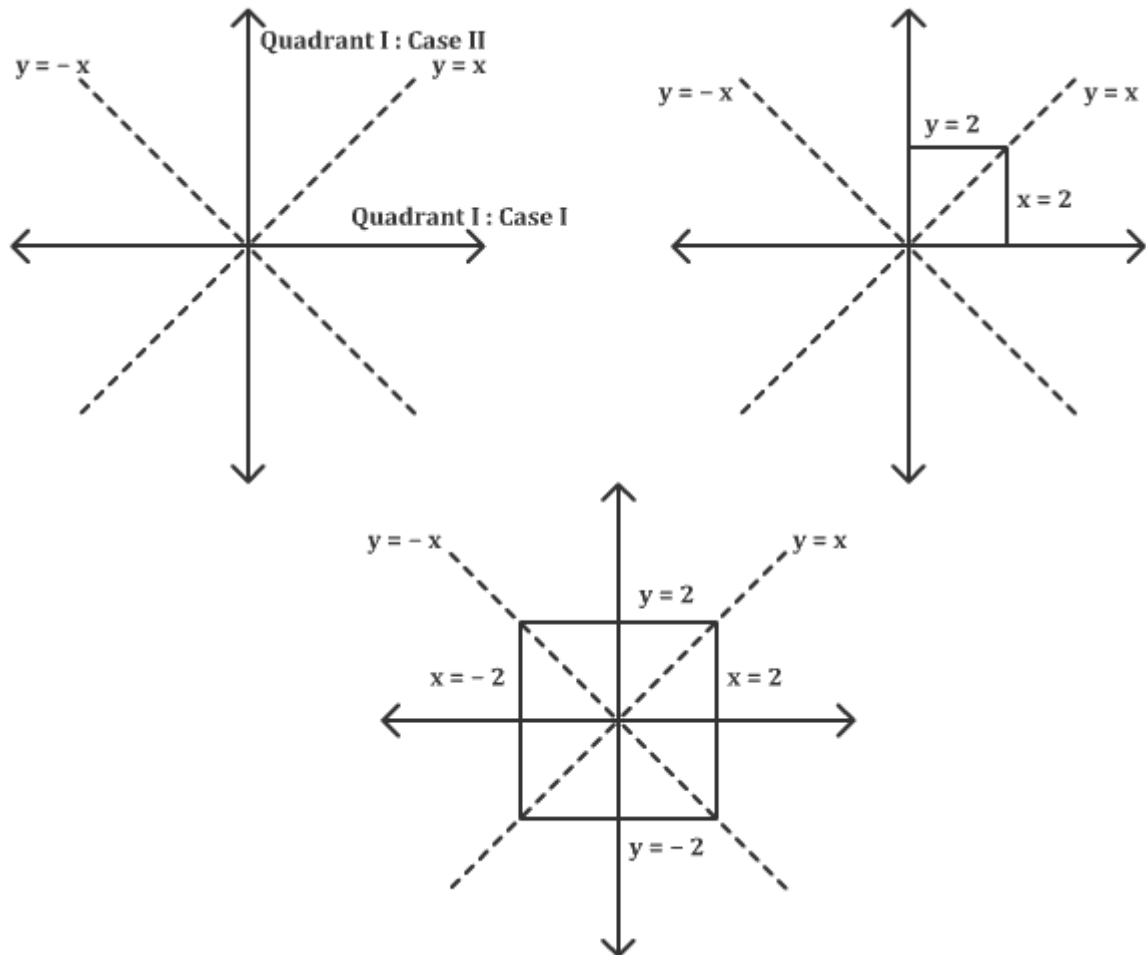


This problem can be solved by trying different ways of placing the tiles on the floor. The maximum number of tiles that can be accommodated is 6 as shown in the figure.



Hence, option 3.

21.



$$|x + y| + |x - y| = 4 \quad \dots (i)$$

Consider the case when  $x$  and  $y$  are both positive. This is the area of quadrant I  
 In this case two cases are possible.

*Case I:*  $x > y$

In this case expression (i) becomes

$$x + y + x - y = 4$$

$$\therefore 2x = 4$$

$$\therefore x = 2$$

*Case II:*  $x < y$

In this case expression (i) becomes

$$x + y + y - x = 4$$

$$\therefore 2y = 4$$

$$\therefore y = 2$$

$\therefore$  The area for the first quadrant is as shown in the figure.

Extending the same logic to other quadrants we get the area as shown in the diagram.

$$\therefore \text{Its area} = 4 \times 4 = 16 \text{ sq. units}$$

Hence, option 3.

**22.**  $AO = OD = 1.5 \text{ cm}$

$$AE + EB = 3 \text{ cm and } AE:EB = 1:2$$

$$\therefore AE = 1 \text{ cm and } EB = 2 \text{ cm}$$

$$OE = AO - AE = 1.5 - 1 = 0.5 \text{ cm}$$

$$\text{Similarly, } NL = 1 \text{ cm, } M = 2 \text{ cm and } OL = 0.5 \text{ cm}$$

OEHL is a square as all its angles are right angles and  $OE = OL$

$$\therefore EH = HL = 0.5 \text{ cm}$$

$$\text{In } \triangle ODL, OD^2 = OL^2 + DL^2$$

$$1.5^2 = 0.5^2 + (0.5 + DH)^2$$

$$2.25 = 0.25 + 0.25 + DH + DH^2$$

$$DH^2 + DH - 1.75 = 0$$

$$DH = \frac{-1 \pm \sqrt{1 - 4(-1.75)}}{2}$$

$$= \frac{(2\sqrt{2} - 1)}{2} \quad (DH > 0)$$

Hence, option 2.

**23.**  $m\angle BCD = m\angle BAC$  and B is common to triangles ABC and CBD.

$\triangle ABC$  is similar to  $\triangle CBD$

$$AB/CB = BC/BD = AC/CD$$

$$AB/12 = 12/9 = AC/6$$

$$AB = 16 \text{ cm and } AC = 8 \text{ cm}$$

$$AD = AB - BD = 16 - 9 = 7 \text{ cm}$$

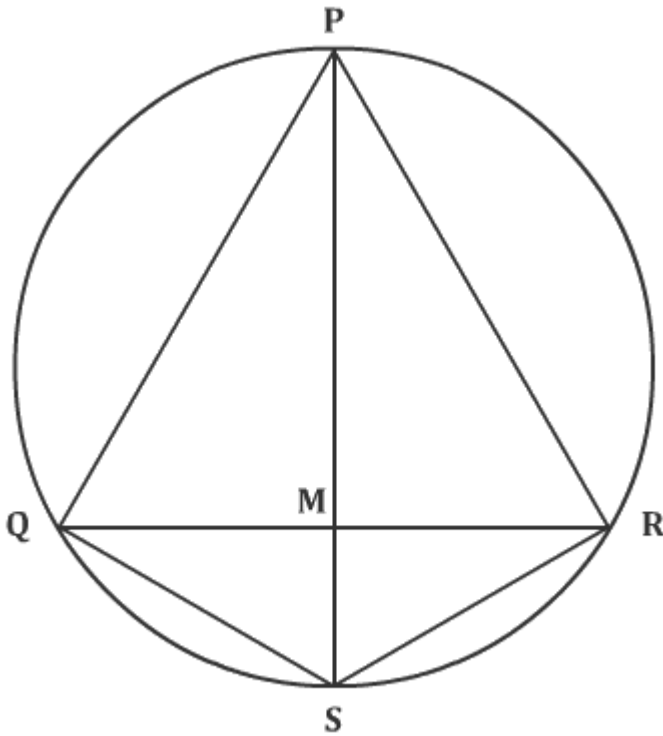
$$\therefore \text{Perimeter of } \triangle ABC = 7 + 6 + 8 = 21 \text{ cm}$$

$$\therefore \text{Perimeter of } \triangle BDC = 9 + 6 + 12 = 27 \text{ cm}$$

$$\therefore \text{Required ratio} = 21/27 = 7/9$$

Hence, option 1.

24.



$\Delta PQR$  is an equilateral triangle and  $PS$  is the diameter.  
 $\therefore m\angle PQS = m\angle PRS = 90^\circ$  (angles subtended in a semi-circle)  
 $PS$  bisects  $QR$  as it is the median of  $\Delta PQR$   
 $m\angle PMQ = m\angle PMR = 90^\circ$   
 $\therefore m\angle QPS = m\angle RPS = 30^\circ$   
 $\therefore m\angle PSQ = m\angle PSR = 60^\circ$

Radius =  $r$   
 $\therefore PS = 2r$   
 As  $\Delta PQS, \Delta PQM, \Delta MQS$  are  $30^\circ-60^\circ-90^\circ$  triangles,  
 $QS = r, PQ = \sqrt{3}r$   
 Similarly,  $RS = r, PR = \sqrt{3}r$   
 $\therefore$  Perimeter of quadrilateral  $PQRS = 2r + 2\sqrt{3}r = 2r(1 + \sqrt{3})$   
 Hence, option 1.

25.  $n$  will be of the form  $11ab$ , where  $a$  and  $b$  are odd numbers.

We are looking for all  $n$ 's divisible by 3.  
 $\therefore 1 + 1 + a + b = 3$  or  $9$  or  $12$  or  $15$  or  $18$   
 $\therefore a + b = 1$  or  $4$  or  $7$  or  $10$  or  $13$  or  $16$   
 $\therefore a + b = 1$  or  $7$  or  $13$  is not possible as the sum of two odd numbers cannot be odd.  
 $\therefore (a, b) = (1, 3), (3, 1), (1, 9), (3, 7), (5, 5), (7, 3), (9, 1), (7, 9), (9, 7)$   
 $\therefore 9$  elements of  $S$  are divisible by 3.

Hence, option 1.









































